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NEW AND OLD PROOFS OF THE PYTHAGOREAN THEOREM.

By **BENJ. F. YANNEY, A. M.**, Mount Union College, Alliance, Ohio, and **JAMES A. CALDERHEAD, B. Sc.**, Curry University, Pittsburg, Pennsylvania.

[Continued from December Number.]

XXII. Let ABC be a triangle, right-angled at C. Assume $CB<CA$. Complete the figure as indicated, the point E being at the middle of AB.

There are three cases: (1) when E is without the circle, (2) in the circumference, (3) within the circle. We treat but one case, when E is within the circle. The other cases may be treated in a similar manner.

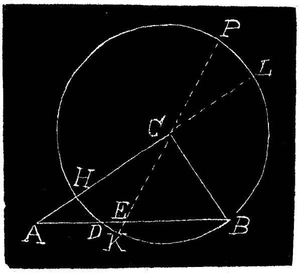

Fig. 17.

$$AD=\frac{AH\cdot AL}{AB}=\frac{b^2-a^2}{c}\ \ldots\ldots\ldots\ldots\ldots\ldots(1).$$

$$DE=\frac{KE\cdot EP}{EB}=\frac{(a-\frac{1}{2}c)(a+\frac{1}{2}c)}{\frac{1}{2}c}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2).$$

Add (1) and (2), $\frac{1}{2}c=\frac{a^2+b^2-\frac{1}{2}c^2}{c}$. $\therefore c^2=a^2+b^2$.

NOTE. This method is credited, by Wipper, to Krueger, 1746. Though in the original, the legs are arbitrarily assumed as unequal, we may pass, by the theory of limits, to the case when the legs are equal.

XXIII. Let ABC be a triangle, right-angled at C. Circumscribe the triangle by a circle. Complete the rectangle. Then,

$$AB\cdot CD=CB\cdot AD+AC\cdot DB,\text{ or } c^2=a^2+b^2.$$

NOTE. Thought this method must have been known independently to many persons, it seems that its first appearance in print was in 1859, in Runkle's *Mathematical Monthly*.

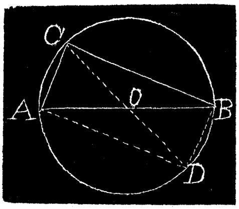

Fig. 18.

XXIV. Let ABC be a triangle, right-angled at C. Construct circles with AC and BC as diameters, respectively. These circles will intersect in AB, as at D.

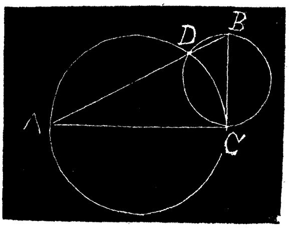

Fig. 19.

Then, $\overline{AC}^2=AD\cdot AB$, and $\overline{BC}^2=BD\cdot AB$.

Add, $\overline{AC}^2+\overline{BC}^2=AB(AD+BD)=\overline{AB}^2$.

NOTE. This is one of Richardson's.

XXV. Let ABC be a triangle, right-angled at C. Complete the figure as indicated.

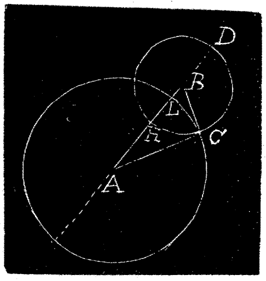

Fig. 20.

Then, $\overline{AC}^2=AH\cdot AD$, and $\overline{BC}^2=BL\cdot BE$.

Add, $\overline{AC}^2+BC=AH\cdot AD+BL\cdot BE$

$=(AB-BC)(AB+BC)+(AB-AC)(AB+AC)$.

$\therefore\ \overline{AB}^2=\overline{AC}^2+\overline{BC}^2$.

Richardson's method is somewhat different. Thus:

$$\overline{AC}^2+\overline{BC}^2=AH(AB+BC)+BL(AB+AC)$$

$$=AB(AH+BL)+AH\cdot BC+BL\cdot AC+BL\cdot AB-BL\cdot AB$$

$$=AB(AH+BL)+AH\cdot BC+BL(BL+2AC)-BL\cdot AB$$

$$=AB(AH+BL)+AH\cdot BH+\overline{BH}^2-BL(AH+BH)$$

$$=AB(AH+BL)+(AH+BH)(BH-BL)$$

$$=AB(AH+BL)+AB\cdot HL=AB(AH+BL+HL)=\overline{AB}^2.$$

XXVI. Fig. 20.

$EH : ED :: HL : DL$. (See Olney, §971). $\therefore\ EH\cdot DL=ED\cdot HL$,

or $(AC+AB-BC)(BC+AB-AC)=(AC+AB+BC)(AC+BC-AB)$.

$\therefore\ \overline{AB}^2=\overline{AC}^2+\overline{BC}^2$.

◆

ARITHMETIC.

Conducted by B. F. FINKEL, Springfield; Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

70. Proposed by J. A. CALDERHEAD, M. Sc., Professor of Mathematics in Curry University, Pittsburg, Pennsylvania.

A owes me \$100 due in 2 years, and I owe him \$200 due in 4 years; when can I pay him \$100 to settle the account equitably, money being worth 6%?